## Key Concepts

### Define:

- **Deductive Reasoning** – A system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.

- **Theorem** – A statement or conjecture that can be proven true by undefined terms, definitions, and postulates. Has been shown to be true.

- **Postulates** – A statement that describes a fundamental relationship between the basic terms of geometry. Accepted as true without proof.

- **Proof** – A logical argument in which each statement you make is supported by a statement that is accepted as true.

- **Two-Column Proof** – A formal proof that contains statements and reasons organized in two columns. Each step is called a statement, and the properties that justify each step are called reasons.

### What are the five essential parts of a good proof?

1. State the theorem or conjecture to be proven.
2. List the given information.
3. If possible, draw a diagram to illustrate the given information. Continue to add to your diagram as you continue to prove different elements of the proof.
4. State what is to be proved.
5. Develop a system of deductive reasoning.
What does a Two-Column Proof look like?

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>The items we include in this portion of our two-column geometric proof will show the progression of our argument. They are the claims we believe to be true.</td>
<td>The items we include in this portion of our two-column geometric proof will explain why the corresponding statements are true. They justify any claims we make.</td>
</tr>
</tbody>
</table>

Step-by-Step Instructions for Writing Two-Column Proofs

1. Read the problem over carefully. Write down the information that is given to you because it will help you begin the problem. Also, make note of the conclusion to be proved because that is the final step of your proof. This step helps reinforce what the problem is asking you to do and gives you the first and last steps of your proof.

2. Draw an illustration of the problem to help you visualize what is given and what you want to prove. Oftentimes, a diagram has already been drawn for you, but if not, make sure you draw an accurate illustration of the problem. Include marks that will help you see congruent angles, congruent segments, parallel lines, or other important details if necessary.

3. Use the information given to help you deduce the preliminary steps of your proof. Every step must be shown, regardless of how trivial it appears to be. Beginning your proof with a good first step is essential to arriving at a correct conclusion.

4. Use the conclusion, or argument to be proven, to help guide the statements you make. Remember to support your statements with reasons, which can include definitions, postulates, or theorems.

5. Once you have arrived at your solution, you may choose to read through the two-column proof you’ve written to be assured that each step has a reason. This helps emphasize the clarity and effectiveness of your argument.

The steps above will help guide you through the rest of the geometry sections you encounter. While they may seem painful and frustrating at times, two-column proofs are extremely helpful because they break things down that seem trivial or intuitive into steps that answer the question “why.”

The first step in a two-column proof is always...

GIVEN
Properties that might help proving geometric concepts.

- Addition Property
- Subtraction Property
- Substitution Property
- Reflexive Property
- Transitive Property

You have solved algebraic equations for a couple years now, but now it is time to justify the steps you have practiced.

List of reasons for each algebraic step:

<table>
<thead>
<tr>
<th>ALGEBRAIC PROPERTIES OF EQUALITY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ADDITION PROPERTY OF EQUALITY</strong></td>
</tr>
<tr>
<td><strong>SUBTRACTION PROPERTY OF EQUALITY</strong></td>
</tr>
<tr>
<td><strong>MULTIPLICATION PROPERTY OF EQUALITY</strong></td>
</tr>
<tr>
<td><strong>DIVISION PROPERTY OF EQUALITY</strong></td>
</tr>
<tr>
<td><strong>DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION or OVER SUBTRACTION</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>SUBSTITUTION PROPERTY OF EQUALITY</strong></td>
</tr>
<tr>
<td><strong>REFLEXIVE PROPERTY OF EQUALITY</strong></td>
</tr>
<tr>
<td><strong>SYMMETRIC PROPERTY OF EQUALITY</strong></td>
</tr>
<tr>
<td><strong>TRANSITIVE PROPERTY OF EQUALITY</strong></td>
</tr>
</tbody>
</table>

If a step requires simplification by combining like terms, write...

SIMPLIFY
Examples

Given: $2x + 3 = 4x - 7$
Prove: $x = 5$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $2x + 3 = 4x - 7$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $3 = 2x - 7$</td>
<td>2. Subtraction property of equality</td>
</tr>
<tr>
<td>3. $10 = 2x$</td>
<td>3. Addition property of equality</td>
</tr>
<tr>
<td>4. $5 = x$</td>
<td>4. Division property of equality</td>
</tr>
<tr>
<td>5. $x = 5$</td>
<td>5. Symmetric property of equality</td>
</tr>
</tbody>
</table>

Given: $5y + 4 = 9$
Prove: $y = 1$

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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>1. $5y + 4 = 9$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $5y = 5$</td>
<td>2. Subtraction property of equality</td>
</tr>
<tr>
<td>3. $y = 1$</td>
<td>3. Division property of equality</td>
</tr>
</tbody>
</table>

Given: $2(8g - 10) = 10g + 4$
Prove: $g = 4$

<table>
<thead>
<tr>
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<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $2(8g - 10) = 10g + 4$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $16g - 20 = 10g + 4$</td>
<td>2. Distributive property of multiplication over subtraction</td>
</tr>
<tr>
<td>3. $6g - 20 = 4$</td>
<td>3. Subtraction property of equality</td>
</tr>
<tr>
<td>4. $6g = 24$</td>
<td>4. Addition property of equality</td>
</tr>
<tr>
<td>5. $g = 4$</td>
<td>5. Division property of equality</td>
</tr>
</tbody>
</table>

Answer to the Essential Question